Cosmological model in 5D, stationarity, yes or no

W. B. Belayev*
Center for Relativity and Astrophysics, box 137, 194355, Sanct-Petersburg, Russia

We consider cosmological model in 4+1 dimensions with variable scale factor in extra dimension and static external space. The time scale factor is changing. Variations of light velocity, gravity constant, mass and pressure are determined with four-dimensional projection of this space-time. Data obtained by space probes Pioneer 10/11 and Ulysses are analyzed within the framework of this model.

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I. INTRODUCTION

Theory, explaining the redshift of the spectra of distant galaxies with variation of time scale factor was put forth by Milne [1]. He also thought the dependence of gravity constant on time t to be possible [2]. Dirac [3] considered gravity constant to be inversely proportional to t, and time dependence of other fundamental constants. However, with the constant length scale factor these theories can not explain the proximity of the density of matter in the Universe to critical value, which follows from Fridman-Robertson-Walker model. At the same time, according to different estimates [4] the cosmological density does not exceed 1/3 of this value.

In this connection Kaluza-Klein theory is of interest. A review of articles about this theory is included in [5]. Internal space is considered to be forming an extra dimension in five-dimensional space-time [5,6]. Models with extra dimensions and possibilities of variation of existing constants are analyzed in [7,8]. Variation of bare constants of nature might be caused by variation of scale factor of internal spaces R . Though presented theories give estimate of rate of relative changing of R orders of magnitude smaller than Hubble constant H, as pointed out Barrow [8], they base on variation of only several constants, leaving other unchanged. Operating on the principle of similarity of processes, having a theory that variation of constants is determined by metric properties of space, one may have another result. However, this assumption needs an additional reasoning.

Dependencies of light velocity c, Planck constant, energy of a particle, its mass, magnitude of force in various coordinate systems in space-time of Minkowsky with cosmic time on scale factor of time N are analyzed in Sec. II A. Redshift dependence of N and correlation between variation of c and H of given space-time are determined in Sec. II B. Sec. III contains analysis of Schwarzschilds metric with cosmic time, presenting dependence of gravitational constant and body motion in gravitational field on N. In Sec. IV energy processes caused by the change of time scale factor are investigated. Radiometric data from Pioneer 10/11 and Ulysses spacecraft are analyzed in Sec. V. Sec. VI A contains solution of Einstein equations for five dimensions $(4+1)$, giving possibility to create a cosmological model with length scale factor which is constant in three-dimensional space and variable in additional space with cosmic time. In Sec. VI B the magnitude of critical density of matter in the Universe corresponding to this model is determined and dependence of redshift on distance is considered.

II. MINKOWSKYS SPACE-TIME WITH COSMIC TIME

A. Mechanics

Lets consider a metric with the line element in four-dimensional space

$$ds^2 = c_0^2 N^2(t)dt^2 - dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2, \tag{1}$$

where r, θ, ϕ are the spherical coordinates, c_0 is the light velocity in a given moment of time t_0, N is the coefficient dependent of t. Assuming $N(t)$ to be slow changing and making the substitution

$$c = c_0 N(t) \tag{2}$$

we get the standard Minkowsky metric. Therefore, light velocity in four-dimensional coordinate system with its center in (t, O), where O is an arbitrary point moving without action of any forces in three-dimensional space is represented by (2). Let us denote

$$\tau = \int_0^t N(t)dt \tag{3}$$

as a time in (t_0, O) coordinate system. The expression tied a distance intervals dr_0 in (t_0, O) system and dr in (t, O) system follows from metrics (1):

$$dl_0 = dl. \tag{4}$$

Then, a velocity v_0 in (t_0, O) coordinate system could be represented as

$$v_0 = \frac{dl_0}{d\tau}. \tag{5}$$

*Electronic address: vladter@ctinet.ru

The energy of a particle for the metric (1) is [9]:

$$E = \frac{m_0 c_0^2 N(t)}{(1 - v_0^2/c_0^2)^{\frac{1}{2}}}, \tag{6}$$

where m_0 is the rest mass of the particle in (t_0, O) coordinate system. Assuming $v_0 = 0$, we get expression for rest energy of the particle

$$E = m_0 c_0^2 N(t). \tag{7}$$

Hence

$$E = E_0 N(t), \tag{8}$$

where $E_0 = m_0 c_0^2$ is the rest energy of the particle in coordinate system with its center in (t_0, O). Detailed discussion of the result (8) will be given in Sec. IV. In view of (2) we obtain from (7) $E = m_0 c^2/N(t)$. It follows herefrom that the rest mass is changing with time

$$m(t) = \frac{m_0}{N(t)}. \tag{9}$$

This variation of mass is relative, i.e, does not occur as variation of mass as number of nucleons.

Let us determine the magnitude of force acting upon the particle. Expression for the vector of the force at small velocities in relation to coordinate system (t, O) is represented by $F = m dV/dt$, where V is the velocity vector. Since (5) the velocity vector in coordinate system (t_0, O) is $V_0 = V/N(t)$, because of (9), (3) the vector of the force is written as

$$F = \frac{m_0}{N(t)} \frac{d\,(V_0 N(t))}{dt} = \frac{m_0}{N(t)} \left[\dot{N}(t) V_0 + \frac{dV_0}{d\tau} N^2(t) \right], \tag{10}$$

where and below overdot denote the derivative with respect to t. Regarding $\dot{N}$ to be small, we get the magnitude of force, acting upon the particle in system (t, O) :

$$f = m_0 \frac{dv_0}{d\tau} N(t) = f_0 N(t), \tag{11}$$

where f_0 is its value in system (t_0, O).

Let us consider variation of Planck constant h with time. The quantum energy in system (t, O) is $E = h/P$, where P is the radiation period. Considering the energy of quantum changing in accordance with (8) we will obtain, taking into account (3):

$$E_0 N(t) = \frac{h N(t)}{P_0(\tau)}, \tag{12}$$

where $P_0(\tau)$ is the radiation period, which is correspondent to energy of quantum, emitted in the moment τ, within coordinate system (t_0, O). As long as $E_0 = h_0/P_0$, where h_0 is the Planck constant, and P_0 is the emission period in system (t_0, O), we may assume h to be constant

$$h = h_0. \tag{13}$$

B. Cosmological parameters

We determine the redshift magnitude for the cosmological model with the metric (1):

$$z = \frac{(\lambda_0(\tau) - \lambda_0(\tau_0))}{\lambda_0(\tau_0)}, \tag{14}$$

where $\lambda_0(\tau)$ is the wavelength of emission, radiated in moment τ in system (t_0, O), $\tau_0 = \tau(t_0)$. This formula gives

$$z = \frac{(P_0(\tau) - P_0(\tau_0))}{P_0(\tau_0)}. \tag{15}$$

Since $P_0(\tau) = PN(t)$ and the period of emission P is constant at any system, bound with the emission time: $P = P_0(\tau_0)$, then in view of $N(t_0) = 1$ we have

$$z = N(t) + 1. \tag{16}$$

Assuming the rate of change $N(t)$ to be constant in time interval $\Delta t = t_0 - t$, we write $z = -\dot{N}(t_0)\Delta t_0$. Denoting

$$H = -\frac{\dot{N}(t_0)}{N(t_0)}, \tag{17}$$

we obtain Hubble Law $z c_0 = H \Delta r_0$, where $\Delta r_0 = \Delta t_0 c_0$.

Let us determine the variation of light velocity per time unit from (2):

$$\dot{c}(t_0) = c_0 \dot{N}(t_0) = -c_0 H \tag{18}$$

Assuming the Hubble constant $H = 50$ km s^{-1} Mpc^{-1} [10,11], we will get $\dot{c}(t_0) = 1.5$ cm s^{-1} yr^{-1}.

Now we consider the temperature change of microwave cosmic background. Since the energy of a quantum of light changes in accordance with (8), the temperature is

$$T = T_0 N(t), \tag{19}$$

where T_0 is the temperature of microwave cosmic background at present.

III. SCHWARZCHILDS METRIC WITH COSMIC TIME

Let us consider the metric

$$ds^2 = \left(1 - \frac{\alpha}{r}\right) N^2(t) dt^2 - \frac{1}{c_0^2} \left[\left(1 - \frac{\alpha}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2 \right], \tag{20}$$

where α is the constant. Assuming $N(t)$ slowly changing with t and the movement to be radial, we solve the equations of the geodesic line [12]:

$$\frac{d}{ds}\left[\left(1-\frac{\alpha}{r}\right)N^2\frac{dt}{ds}\right]=0 \quad (21)$$

$$\left(1-\frac{\alpha}{r}\right)N^2\left(\frac{dt}{ds}\right)^2-\frac{1}{c_0^2}\left(1-\frac{\alpha}{r}\right)^{-1}\left(\frac{dr}{ds}\right)^2=1. \quad (22)$$

Integrating (21) we obtain

$$\frac{dt}{ds}=\beta\left(1-\frac{\alpha}{r}\right)^{-1}N^{-2}, \quad (23)$$

where β is the integration constant. We substitute this equation into (22):

$$\left(1-\frac{\alpha}{r}\right)^{-1}N^{-2}-\frac{1}{c_0^2}\left(1-\frac{\alpha}{r}\right)^{-3}N^{-4}\left(\frac{dr}{dt}\right)^2=\beta^{-2}. \quad (24)$$

If movement is such that particle can reach infinite values, then

$$\beta^{-2}=N^{-2}-\frac{1}{c_0^2}N^{-4}v_i^2, \quad (25)$$

where v_i^2 is the velocity of particle at infinity. Assuming $v_i^2N^{-2} \ll c_0^2$, we get $\beta^2 = N^2$. Then, equation (24) is written as

$$\left(\frac{dr}{dt}\right)^2=c_0^2\left(1-\frac{\alpha}{r}\right)^2\frac{\alpha}{r}N^2. \quad (26)$$

Differentiating this expression by t and taking into account small magnitude of α/r and slow variation of $N(t)$, we obtain

$$\frac{d^2r}{dt^2}=-\frac{\alpha c_0^2N^2}{2r^2}. \quad (27)$$

At $t = t_0$ this equation express Newtons Law for radial movement of particle in external gravitational field of a spherical mass, if $\alpha = 2G_0M_0/c_0^2$, where G_0 is the gravity constant in system (t_0, O), M_0 is spherical mass in this coordinate system. Thus, (27) is presented as

$$\frac{d^2r}{dt^2}=-\frac{G_0M_0N^2}{2r^2}. \quad (28)$$

Because of (9) $M = M_0/N$ the expression, determining variation of gravity constant G with time in formula for Newtons Law $d^2r/dt^2 = -GM/r^2$, is given by

$$G=G_0N^3. \quad (29)$$

Now we change to (t_0, O) coordinate system in expression (28). Since values of distance and time intervals in this system are $dr_0 = dr$ and $d\tau = N(t)dt$, we have

$$\frac{d^2r}{dt^2}=-\frac{d}{dt}\left(\frac{dr_0}{d\tau}N(t)\right)=N^2(t)\frac{d^2r_0}{d\tau^2}+\dot{N}(t)\frac{dr_0}{d\tau}. \quad (30)$$

Assuming second term in the right side of equation to be small, we get

$$\frac{d^2r_0}{d\tau^2}=-\frac{G_0M_0}{2r_0^2}. \quad (31)$$

Since right side of this formula determines force acting upon a body with unity mass at the distance r_0, orbit of the body, moving in external gravity field does not change within the (t_0, O) coordinate system. Absence of dependence of a length element on choice of time zero means constancy of shape and size of the orbit.

IV. ENERGY PROCESSES AND GENERATION OF MATTER

We define the energy change (7) with time

$$W=\frac{dE}{dt}=m_0c_0^2\dot{N}(t). \quad (32)$$

In view of (2) and (9) we can write $W = \frac{\dot{N}}{N}mc^2$, and, as follows from (17), at present time

$$W=-Hm_0c_0^2. \quad (33)$$

Thus, energy of a body with mass 1 kg decreases by 0.14 J every second. Since, as follows from energy conservation law, it does not disappear, it means that the energy is liberated in some way. This conclusion, so it should seem, rules out the theory considered. In truth, according to (33) the Sun should liberate 2.8×10^{29} J/sec, while it actually does 3.8×10^{26} J/sec.

However, Bondi and Gold [13], Hoyle [14], Jordan [15] have put forth a hypothesis of generation of matter within the framework of expanding Universe model. Analysis of currents of helium, coming out of depths of the Earth, testifies about its radiogenic origin [16]. On the basis of these data, the hypothesis of generation of matter within the Earth gains further development [17]. However, to prove this theory, extra arguments are necessary.

Lets determine, what quantity of matter could emit in the form of additional nucleons in a unit of time, if all energy (32) is used for its generation. We consider the change of mass $m_0(t)$ resulting from generation of matter in moment of time $\tau(t)$ in coordinate system (t_0, O). This mass emits per one time unit (32):

$$U=-W=-\dot{N}(t)m_0(t)c_0^2. \quad (34)$$

On the other hand, formula (7) yields

$$U=\dot{m}_0(t)N(t)c_0^2. \quad (35)$$

where $\dot{m}_0$ is the mass, generated per time unit in system (t_0, O). Solving equation $\dot{m}_0(t)/m_0(t) = -\dot{N}(t)/N(t)$ with initial conditions $m_0(t_0) = 1$, $N(t_0) = 1$, we obtain

$$m_0(t)=\frac{m_0}{N(t)} \quad (36)$$

and, at present time,

$$\dot{m}_0(t_0) = Hm_0. \tag{37}$$

It should be pointed out, however, it is not essential, that all emitted energy is used for generation of matter, but in several cases, probably, is released in some other forms, for instance, as heat.

V. ANALYSIS OF EXPERIMENTAL DATA

During radiometric analysis of Pioneer 10/11 spacecrafts data [18], additional acceleration $a_r \approx 8.5 \times 10^{-8}$ cm/s^2, directed towards the Sun was detected. This value is on the verge of limits, determined by error estimate of observed acceleration of Ulysses [18]. According to result, obtained in Sec. III (31), trajectory of a body does not change with time. We consider possible explanation of presence of quasi acceleration. Detecting change of the wavelength of a received wave λ could be a resultant of two factors: decrease of the light velocity and variation of the cycle of radiation of the spacecraft generator P_g in the scale of receiving equipment:

$$\dot{\lambda} = \dot{c}P_g + c\dot{P}_g\ . \tag{38}$$

Variation of the cycle of radiation could be related to dependence of processes, which determine work of generator and receiving equipment, on time scale factor. This problem needs further consideration. If we make assumption $\dot{P}_g(t_0) = -P_g(t_0)H$, then in view of (18) we can write $\dot{\lambda}(t_0) = -2P_g(t_0)H$. The rate of the relative change of wavelength is

$$\frac{\dot{\lambda}(t_0)}{\lambda} = -2H. \tag{39}$$

At the same time it is presented as $\dot{\lambda}/\lambda = a_r/c$, in assumption cycle of radiation and light velocity are constant, and variation of wavelength is caused by additional acceleration. Hence (39) one can get a value of $H = 43.7$ km s^{-1} Mpc^{-1}, which is in agreement with its estimate [19,11].

If P_g does not change, then the rate of the relative change of wavelength is

$$\frac{\dot{\lambda}(t_0)}{\lambda} = -H, \tag{40}$$

This one gives a value of $H = 87.4$ km s^{-1} Mpc^{-1}.

VI. COSMOLOGICAL MODEL IN FIVE DIMENSIONS

A. Partial solution of Einsteins equations in five dimensions

We consider the solution of the Einsteins equations in five dimensions [5]:

$$\hat{R}^{\alpha\beta} - \frac{1}{2}g^{\alpha\beta}\hat{R} = 8\pi G T^{\alpha\beta}, \tag{41}$$

where $T^{\alpha\beta}$ is the matter energy momentum tensor, $\hat{R}^{\alpha\beta}$ is the Ricci tensor, $\hat{R}$ is the scalar curvature of space, $g^{\alpha\beta}$ is the metric tensor components. We assume that parameters such as gravity constant, mass, light velocity and pressure are slowly changing with time in accordance with results, obtained in Sec. II, III. We look for solution in the form

$$ds^2 = N^2(t,\psi)dt^2 - \frac{1}{c_0^2}\left[dx^{12} + dx^{22} + dx^{32} + R^2(t,\psi)d\psi^2\right], \tag{42}$$

where $x^1,\ x^2,\ x^3$ is the space coordinates of the four-dimensional space-time, ψ is the coordinate of extra dimension, created by compactified internal space, $N(t,\psi)$, $R(t,\psi)$ are the arbitrary functions t and ψ. This metrics reduces to (1) metrics at $\psi = const$. Lets denote derivatives with respect to ψ by $(*)$ and $x^0 \equiv t$, $x^4 \equiv \psi$. So the nonzero components of a tensor $\hat{R}^{\alpha\beta}$ are

$$\begin{aligned} \hat{R}^{00} &= -\tfrac{\ddot{R}}{RN^4} + c_0^2\tfrac{N^{**}}{R^2N^3} + \tfrac{\dot{N}\dot{R}}{RN^5} - c_0^2\tfrac{N^*R^*}{R^3N^3}, \\ \hat{R}^{44} &= -c_0^4\tfrac{N^{**}}{NR^4} + c_0^2\tfrac{\ddot{R}}{N^2R^3} + c_0^4\tfrac{R^*N^*}{NR^5} - c_0^2\tfrac{\dot{R}\dot{N}}{N^3R^3}. \end{aligned} \tag{43}$$

Value of the scalar curvature of the space turns out to be

$$\hat{R} = g_{00}\hat{R}^{00} + g_{44}\hat{R}^{44} = -2\frac{\ddot{R}}{RN^2} + 2c_0^2\frac{N^{**}}{NR^2}. \tag{44}$$

The nonvanishing equations (41) are

$$\begin{gathered} \tfrac{\dot{N}\dot{R}}{RN^5} - c_0^2\tfrac{N^*R^*}{R^3N^3} = 8\pi G T^{00} \\ c_0^4\tfrac{R^*N^*}{NR^5} - c_0^2\tfrac{\dot{R}\dot{N}}{N^3R^3} = 8\pi G T^{44} \\ c_0^2\tfrac{\ddot{R}}{RN^2} - c_0^4\tfrac{N^{**}}{NR^2} = 8\pi G T^{ii},\ i = 1,2,3. \end{gathered} \tag{45}$$

Hereinafter we will assume N and R independent from ψ:

$$N^* = 0,\ R^* = 0. \tag{46}$$

We consider a five-dimensional energy tensor with nonzero diagonal matrix elements look like

$$T_i^j := \begin{pmatrix} \rho & . & . & . & . \\ . & \frac{p_{ext}}{c^2} & . & . & . \\ . & . & \frac{p_{ext}}{c^2} & . & . \\ . & . & . & \frac{p_{ext}}{c^2} & . \\ . & . & . & . & \frac{p_{int}}{c^2} \end{pmatrix}, \tag{47}$$

where ρ is the density of matter in the Universe, p_{ext} is the isotropic pressure in external three-dimensional space, p_{int} is the pressure in additional dimension. We write the energy tensor [12,9]:

$$T^{ij} = \left(\rho + \frac{p^{ij}}{c^2}\right) u^i u^j - g^{ij}\frac{p^{ij}}{c^2}, \tag{48}$$

where $p^{ij} = 0,\ i \neq j$; $p^{ii} = p_{ext},\ i = 0...3$; $p^{44} = p_{int}$, and $u^i = dx^i/ds$ are the components of velocity vector of the fluid elements. Assuming [12,21] $u^i = 0,\ i = 1...4$, from (42) we obtain

$$u^0 = N^{-1}. \tag{49}$$

We present nonzero components of the energy tensor

$$\begin{aligned} T^{00} &= \rho N^{-2} \\ T^{ii} &= c_0^2 c^{-2} p_{ext},\ i = 1,2,3 \\ T^{44} &= c_0^2 c^{-2} p_{int} R^{-2}. \end{aligned} \tag{50}$$

Now we change over to coordinate system (t_0, O). Since an element of length does not change at that, in view of (9), $\rho = \rho_0(t)N^{-1}$, where $\rho_0(t)$ is the density of matter in (t_0, O) system in time $\tau(t)$. A density $\rho_0(t)$ can decrease as a result of transformation of matter into electromagnetic radiation. When assuming a possibility of generation of matter (36), its accretion is larger then its decrease in the result of emission of energy, and therefore at $t \leq t_0$:

$$\rho_0 N^{-1}(t) \leq \rho_0(t) \leq \rho_0, \tag{51}$$

where $\rho_0 \equiv \rho_0(t_0)$. From (11) follows that $p = p_0 N(t)$, where p and p_0 are the pressures in (t, O) and (t_0, O) coordinate systems accordingly. Thus, taking into account formula for the light velocity (2), the expressions (50) can be written as

$$\begin{aligned} T^{00} &= \rho N^{-3} \\ T^{ii} &= p_{0ext} N^{-1},\ i = 1,2,3 \\ T^{44} &= p_{0int} N^{-1} R^{-2}. \end{aligned} \tag{52}$$

Then, given formula of the variation of the gravity constant (29), the equations (45) with conditions (46) are equal

$$\dot{N}\dot{R}N^{-5}R^{-1} = 8\pi G_0 \rho_0(t) \tag{53}$$

$$-c_0^2 \dot{R}\dot{N}R^{-1}N^{-5} = 8\pi G_0 p_{0int} \tag{54}$$

$$c_0^2 \ddot{R}N^{-4}R^{-1} = 8\pi G_0 p_{0ext}\ . \tag{55}$$

Let us denote

$$K = 8\pi G_0 \rho_0 \tag{56}$$

and find several solutions (55), which have physical meaning for the case of possible generation of matter $\rho_0(t) = \rho_0/N(t)^d$. Lets assume that

$$N = R\ . \tag{57}$$

Then, given generation of matter, the equation (53) transforms into form $\dot{N}^2 N^{2D} = K$, where $D = d/2 - 3$. If $D \neq 1$, its solution is

$$N(t) = \left[1 - (D+1)\sqrt{K}(t - t_0)\right]^{1/(D+1)}. \tag{58}$$

Denoting $\hat{t} = t_0 - t$ we obtain

$$\hat{N}(\hat{t}) \equiv N(t(\hat{t})) = \left[1 - (D+1)\sqrt{K}\hat{t}\right]^{1/(D+1)}. \tag{59}$$

Now we change over to coordinate system (t_0, O) by denoting

$$\hat{\tau} = \int_0^{\hat{t}} \hat{N}(\hat{t})d\hat{t}. \tag{60}$$

Then (59) is rewritten as

$$N_\tau(\hat{\tau}) \equiv \hat{N}(\hat{t}(\hat{\tau})) = \left[1 + (D+2)\sqrt{K}\hat{\tau}\right]^{1/(D+2)}. \tag{61}$$

B. Cosmological parameters

The mean value of density of matter in the Universe, obtained by different techniques [4] with error $50 - 75\%$, comprises about 1/5 of critical density, defined with help of the Fridman-Robertson-Walker model, $\rho_{crit} = 3H^2/(8\pi G_0)$. The field equation (53) gives at $t = t_0$:

$$8\pi G_0 \rho_0 = \dot{N}(t_0)\dot{R}(t_0) \tag{62}$$

or in view of (17):

$$8\pi G_0 \rho_0 = -H\dot{R}(t_0). \tag{63}$$

Assuming that

$$\dot{N}(t_0) = \dot{R}(t_0) \tag{64}$$

we obtain the density value

$$\rho_0 = \frac{1}{8}\frac{H^2}{\pi G_0}. \tag{65}$$

which is within the limits of the density of matter in the Universe, derived from measurements. It is natural to assume, that if equality of rate of change of the time scale factor and length of internal space (64) takes place at present, it is fulfilled permanently.

Let us determine dependence between the magnitude of redshift and the distance. The distance to the light source is $r = c_0\hat{\tau}$. From (56) and (65) follows that $K = H^2$. Then, in view of (16) and (59), (61) we obtain

$$z = \left[1 + (D+2)\frac{Hr}{c_0}\right]^{1/(D+2)} - 1. \qquad (66)$$

If $D > -1$, then this result gives the picture to be analogous to accelerated expansion in the expanding Universe model.

VII. CONCLUSION

Value of density of matter in the Universe, determined with help of cosmological model in five dimensions with length scale factor which is constant in external space and changing in internal space and variable time scale factor, is in agreement with observation data. An apparent additional acceleration, indicated from Pioneer 10/11 and Ulysses data, does not run counter to this model. At the same time, conclusion about energy emission and generation of matter as a result of decrease of time scale factor needs additional experimental confirmation.

Thus, these results testify about possibility of length time factor to be stationary in the significant part of the observed Universe.

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